Statistics

EXAMPLES 1 — background, normal distribution

Q1. Given that a variable *X* is distributed normally with mean value 23.5 and standard deviation 3.0, find the probability that:

      (a) X < 28
      (b) X < 17
      (c) X > 30
      (d) 26 < X < 28
      (e) X differs from its mean by more than 6.0

Q2. IQ scores are derived in such a way that the mean for the population is 100 and the standard deviation is 15. In a population of 60 million, how many have IQs

      (a) >145
      (b) <80
      (c) within one standard deviation of the mean?

Q3. African meerkats (*Suricata suricatta*) have a mean height of 30 cm with a variance of 4 cm. Their heights are normally distributed.

      (a) What is the standard deviation of the heights of African meerkats?
      (b) What proportion of meerkats are between 30 and 31 cm tall?
      (c) If you took a thousand randomly-selected meerkats, how many would you expect to be shorter than 28.5 cm?
      (d) What are the 95% confidence intervals for meerkat heights (the heights, centred around the mean, within which 95% of meerkat heights fall)?
      (e) Pilchard the meerkat is 33 cm tall. What is the probability that a meerkat of Pilchard's height (or greater) could come from the population of African meerkats?
      (f) What is the probability that a meerkat whose height is less than Pilchard's could come from this population?
      (g) What is the approximate probability that a meerkat whose height is *exactly* that of Pilchard's could come from this population?

Q4. A researcher reporting the results of a functional imaging study states that blood flow in the left cerebellum decreased while subjects thought about music. The researcher calculated this change to be equivalent to a *Z* score of –2.4.

      (a) What is the probability that this *Z* score (or one still more extreme in the same, negative, direction) could have arisen by chance?
      (b) What is the probability that a *Z* score of ≥ +2.4 or ≤ –2.4 could have arisen by chance?
      (c) If mean left cerebellar blood flow is 50 ml per minute with a standard deviation of 5 ml per minute while the subjects were resting, what was the left cerebellar blood flow while the researcher's subjects were thinking about music?
      (d) If the researcher had simultaneously scanned 100 areas of the brain and calculated *Z* scores for each of them (by comparing 'music' blood flow to 'resting' blood flow in each case), what is the probability of obtaining at least one *Z* score at least as extreme as ±2.4 if listening to music did not in fact affect brain blood flow at all?

Q5.   A traveller one day was making his way
        through a woods both wild and deep.
    The road split in two and he muttered, '*Mon Dieu*!
        Where in the world shall I sleep?'
    To the south lay an inn most noted for sin,
        but a constable shrewd and upright.
    To the north (said the sign), a village benign,
        a safer abode for the night.
    The village, it seemed, was a traveller's dream,
        for thieves chose the inn, as a rule,
    But the sheriff in town had earned wide renown
        as a lax and incompetent fool.

Suppose that the probability of being robbed is .60 at the inn and .20 in the village. On the other hand, the probability that the constable at the inn will recover the traveller's money is .70, but the probability that the sheriff in the village will recover the traveller's money is only .10. Which is the better choice?