Answers to Examples 1 — background and normal distribution

(Answers calculated by RNC — *caveat emptor*. Thanks to MRFA for checking them.)

Q1 RV (a) 0.933; (b) 0.015; (c) 0.015; (d) 0.136; (e) 0.046

Explanations...

If X is a normally-distributed random variable with mean 23.5 and SD 3.0, then we can invent a variable Z that has a mean of 0 and a SD of 1 - a 'standard' normal variable' by calculating Z = (X - mean)/SD = (X - 23.5)/3.0.

So when we want to ask 'what's the probability that X is less than 28', we can ask instead 'what's the probability that Z is less than (28 - 23.5)/3.0 = 1.5'. We can look up the probability of Z being less than 1.5 from tables of Z; it's 0.933. So this is also the probability that X < 28.

If you want to find the probability that X > 30, that's equivalent to asking 'what's the probability that Z > (30 - 23.5)/3.0 = 2.167'. From tables, the probability that Z is **less** than 2.167 is 0.985, so the probability that Z is **bigger** than 2.167 (or X is bigger than 30) is 1 - 0.985 = 0.015.

This logic applies to all these examples. When you want to find the probability that 26 < X < 28, find the probability that X < 28, and take away from it the probability that X < 26.

- Q2 IQ (Since you're multiplying probabilities by a large number 60,000,000 you will notice differences between the answers you'd get from your tables and those you'd get with a computer. I'd expect you to use the tables you'll have to in the exam but have quoted both answers here.)
 - (a) 78,000. The probability P(IQ > 145) is the same as the probability P(Z > 3), which is 0.0013 from your tables. So this corresponds to $0.0013 \times 60,000,000 = 78,000$ people. (If you calculate this more precisely with a computer, you get a probability of 0.001349967... and the answer 80,988.)
 - (b) P(Z < -1.33) = 0.0918, so the answer's 5,508,000 (or, with a computer, 5,472,677).
 - (c) P(-1 < Z < 1) = 0.6826, so the answer's 40,956,000 (or, with a computer, 40,961,369).

Q3 meerk (a) SD = 2 cm; (b) P(0 < Z < 0.5) = 0.192; (c) 227; (d) 26.08 to 33.92 cm; (e) 0.067; (f) 0.933; (g) zero.

To explain (g) a little... the probability of finding a meerkat whose height is the same as a particular value depends on what we mean by 'the same as'! As we become more and more restrictive (the meerkat has to be within a centimetre... millimetre... micron... of the specified height) the probability of finding such a meerkat becomes smaller and smaller. As the range of acceptable heights shrinks to zero, so does the probability, so the probability of finding a meerkat of 'exactly' a given height is zero.

Q4 RCBF (a) 0.0082; (b) 0.0164; (c) 38 ml/min; (d) 0.809.

Explanation of (d): a Z score of ± 2.4 is equivalent to p = 0.0164, so P(make at least one Type I error at p = 0.0164) = 1 - P(never make any Type I errors at <math>p = 0.0164 in 100 comparisons) = $1 - (1-0.0164)^{100}$ = 1 - 0.191 = 0.809.

Caveat: the method for finding (d) doesn't take into account the fact that nearby areas are likely to have related blood flows — but then this is a statistics example, not an functional imaging tutorial.

Q5 poem Depends on the food, accommodation and risk aversion; both P(lose money) = 0.18