NST 1B Experimental Psychology
Statistics practical 1

Correlation and regression

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Everything (inc. slides) also at
pobox.com / ~rudolf / psychology

Have you read the *Background Knowledge* (§1)?

Did you remember to bring:

- the stats booklet
- your calculator
- your data from the last practical **(mental rotation)?**

If not…

oops!

Plan of this session:

- we'll cover the ideas and techniques of correlation and regression. (The handout covers everything you need to know and more: Section 2 for today.)

- you can analyse your own data (± have a go at the examples)

- you can ask questions (about this practical, the background material, or anything statistical) and we'll try to help.

- Afterwards, **practise.**

Remember:

Wavy-line stuff in the handout is for reference only.

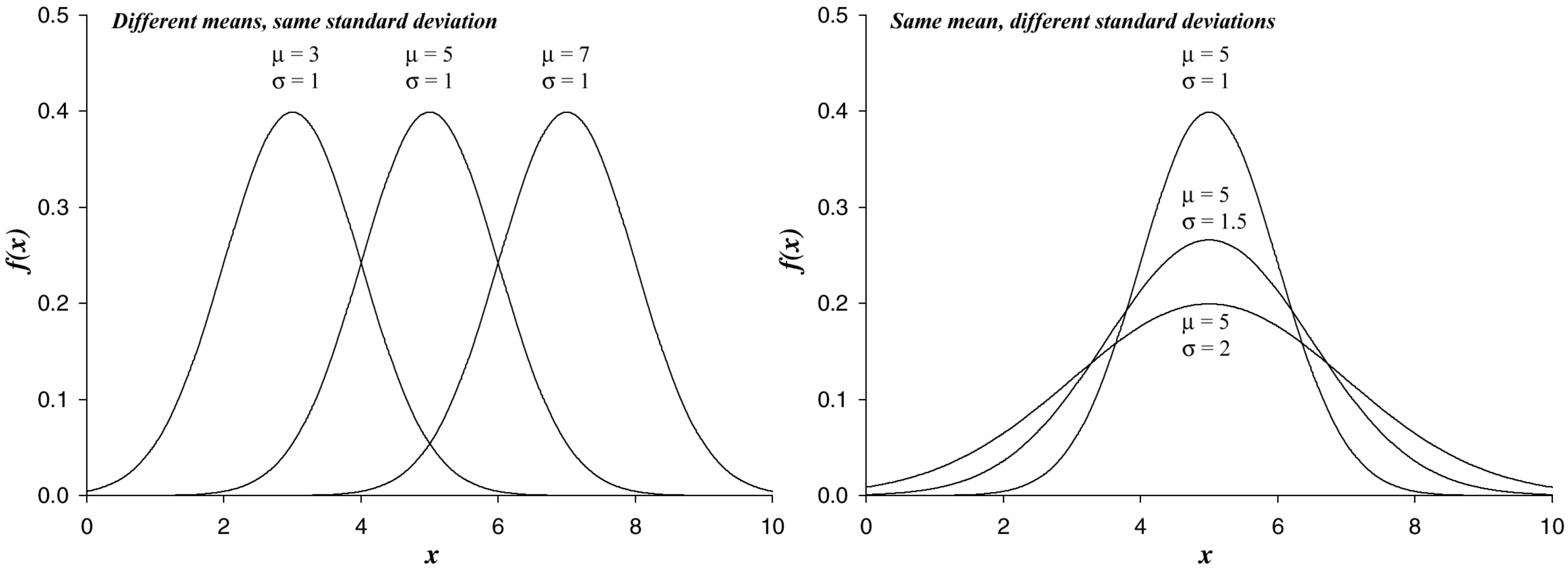
You might be interested.

You might refer to it in the future.

You do not need to understand or learn it.

You should already know (from NST 1A or booklet §1)…

- measures of **central tendency** (e.g. mean, median, mode)
- measures of **dispersion** (e.g. variance, standard deviation)
- histograms and distributions
- the logic of null hypothesis testing



You should already know (from NST 1A or booklet §1)…

mean

$$\bar{x} = \frac{\sum x}{n}$$

sample variance

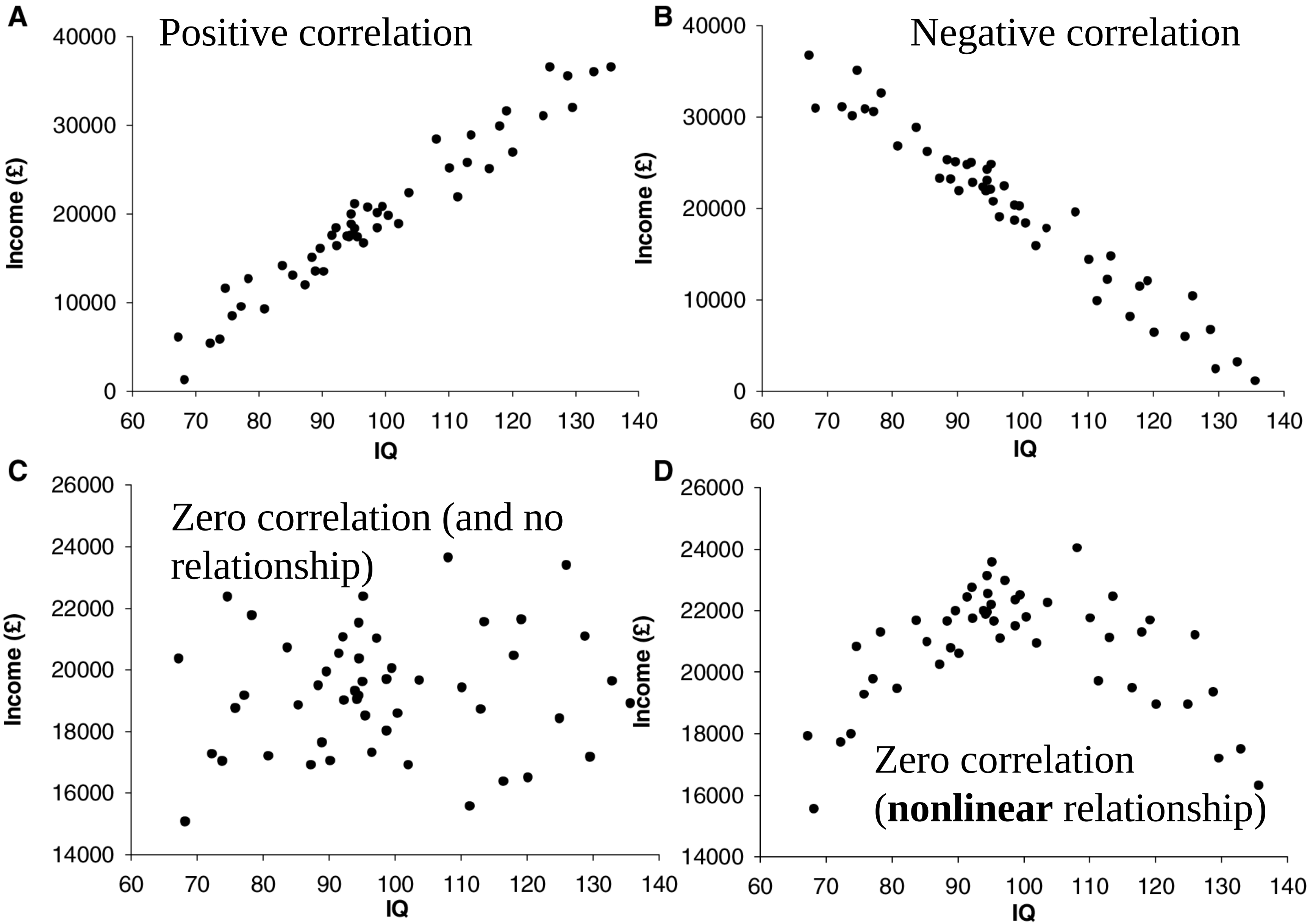
$$s_X^2 = \frac{\sum (x - \bar{x})^2}{n-1} \left(= \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} \right)$$

sample standard deviation (SD)

$$s_X = \sqrt{s_X^2} = \sqrt{\frac{\sum (x - \bar{x})^2}{n-1}} \left(= \sqrt{\frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1}} \right)$$

The relationship between two variables

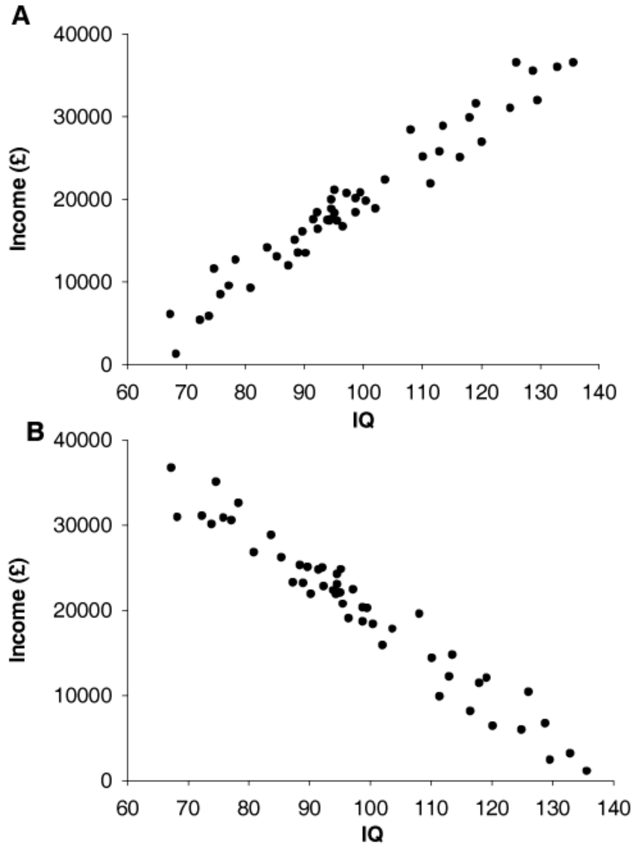
Scatter plots show the relationship between two variables



Correlation

There are no amusing or attractive pictures to do with correlation anywhere in the world.

The covariance measures how much two variables vary together. Good name, eh?



Sample covariance:

$$\mathrm{cov}_{XY} = \frac{\sum (x-\bar{x})(y-\bar{y})}{n-1}$$

There's another formula (below) that's easier to use if you have to calculate the covariance by hand. But you shouldn't need to if you can work your calculator, because... *(next slide…)*

$$\mathrm{cov}_{XY} = \frac{\sum (x-\bar{x})(y-\bar{y})}{n-1} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{n-1}$$

r, the Pearson product–moment correlation coefficient

The covariance is big and positive if there is a strong positive correlation, big and negative if there is a strong negative correlation, and zero if there's no correlation. But how big is 'big'? That depends on the standard deviations (SDs) of X and Y, which isn't very helpful. So instead we calculate *r*:

$$r_{XY} = \frac{\mathbf{COV}_{XY}}{s_X s_Y}$$

r varies from –1 to +1. **Your calculator calculates *r*.**
***r* does not depend on which way round *X* and *Y* are.**

r — WORKED EXAMPLE: mental rotation

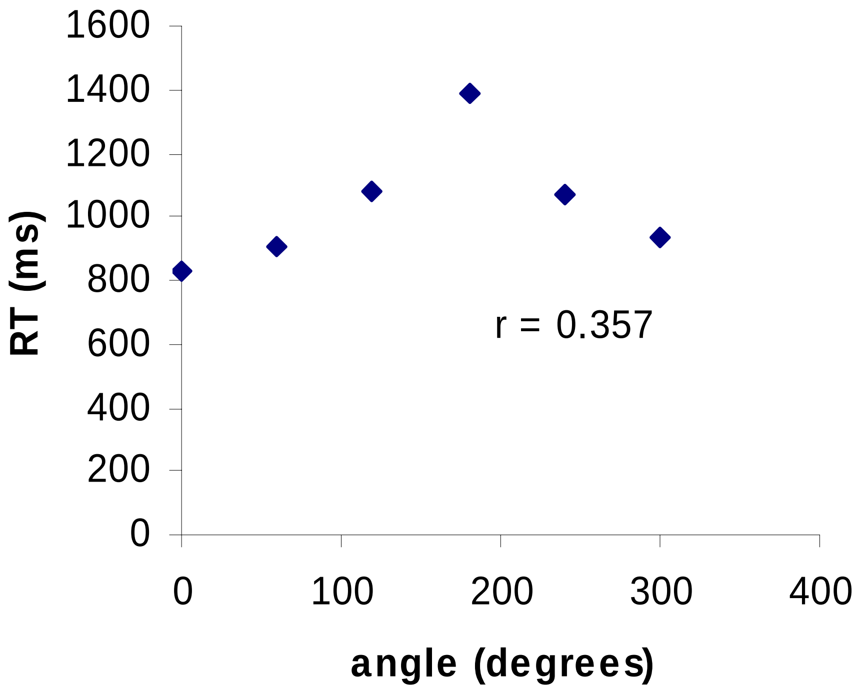
Work through this one now with your calculator.
We'll pause for a moment to do that...

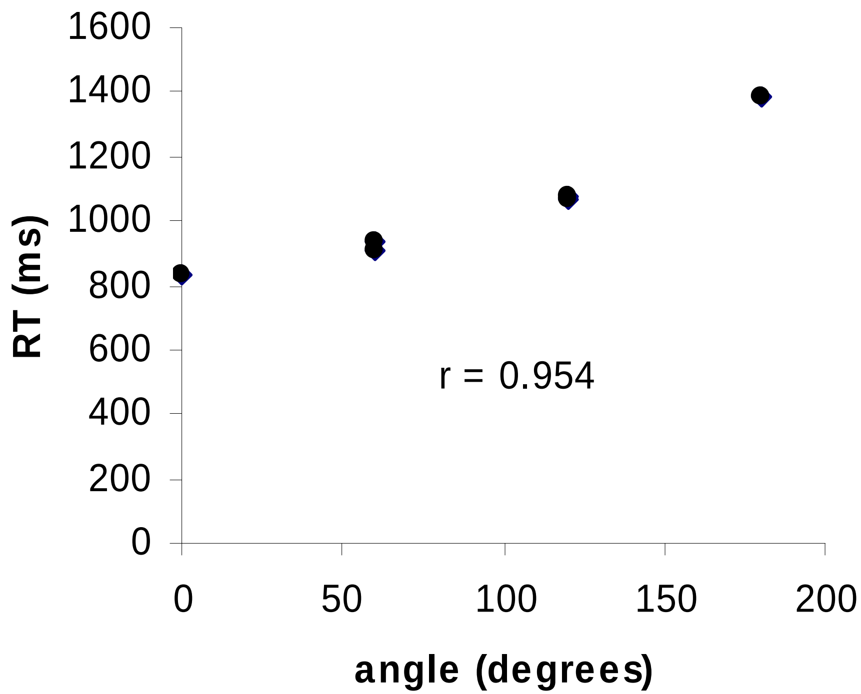
Angle	0	60	120	180	240	300
Group mean RT (ms, to 0 d.p.) (simplest task)	830	908	1079	1387	1070	935

For correlation (r):

	Casio fx115s	Other Casio models
Enter linear regression (LR) mode	MODE 3	MODE →REG→Lin
Clear the stats memory	SHIFT C (Scl)	SHIFT AC (Scl) =
Enter values of x, y pairs (e.g. $x = 53$, $y = 17$)	5 3 [(--- (x_D, y_D) 1 7 M+ (DATA DEL) etc.	5 3 , 1 7 M+ etc.
Read out desired coefficients *(see keypad and inside lid)* r	SHIFT 9 (r)	SHIFT ((r) =
n	RCL 3 ($x\sigma_{n-1}$)	RCL hyp (C)

r — WORKED EXAMPLE: mental rotation



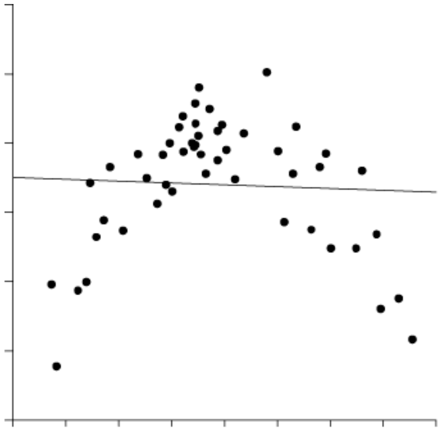


Oops.
Not very linear.

Shortest angle (so ‘240°’ becomes 120°; ‘300°’ becomes 60°).
Much more linear.

Never give up! Never surrender! Never forget...

‘Zero correlation’ doesn’t imply ‘no relationship’.



So always draw a scatter plot.

Correlation does not imply causation.

Correlation, causation… A real-world example

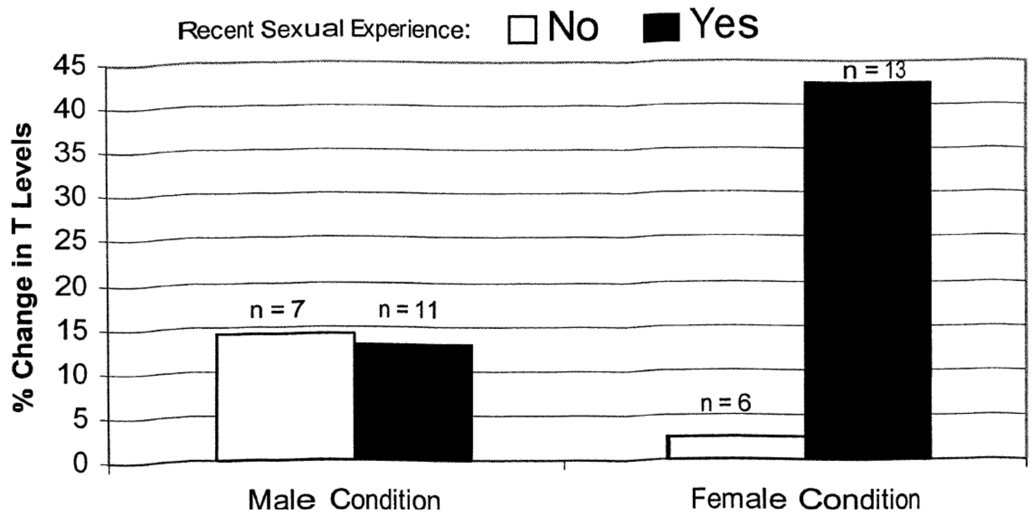
J.R. Roney et al. / Evolution and Human Behavior 24 (2003) 365–375

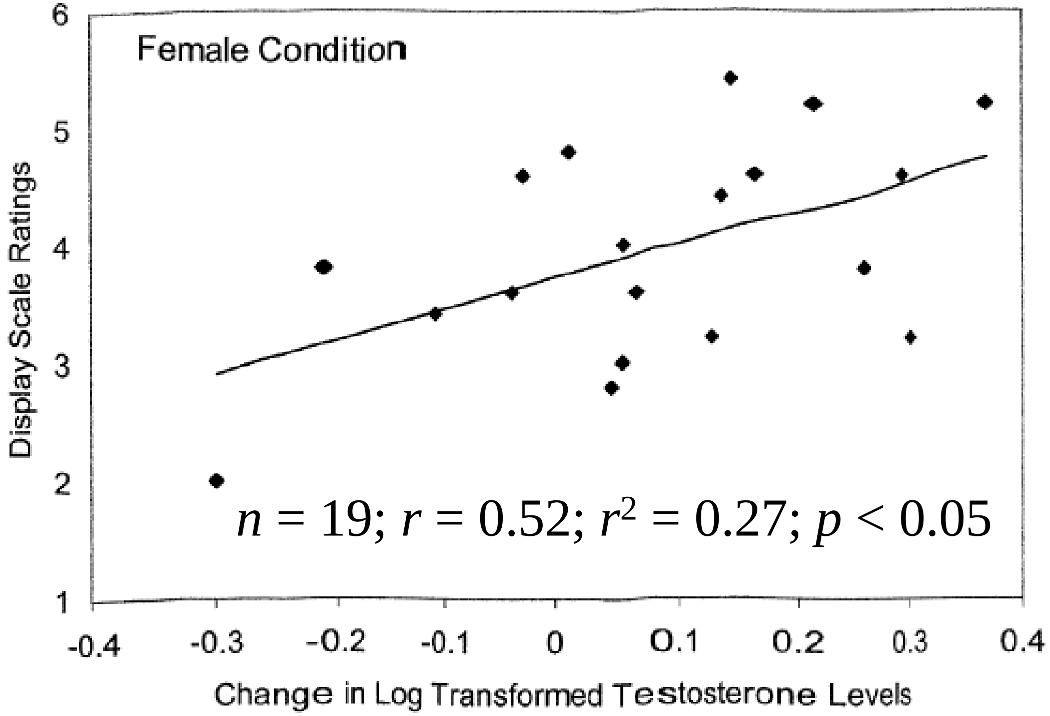
- Male college students (18–36 years old) engaged in 5 min conversation with a stooge, who was either **male** (23 or 32 y.o.) or **female** (19–23 y.o.). Didn't know that this was part of an experiment.

- Saliva samples taken before and after conversation. Testosterone (T) measured.

- 'Recent sexual experience' = current relationship or sex in last 6 months.

- Stooge rated how much they thought the subject was 'displaying' to them (e.g. talkative, showed off, tried to impress).

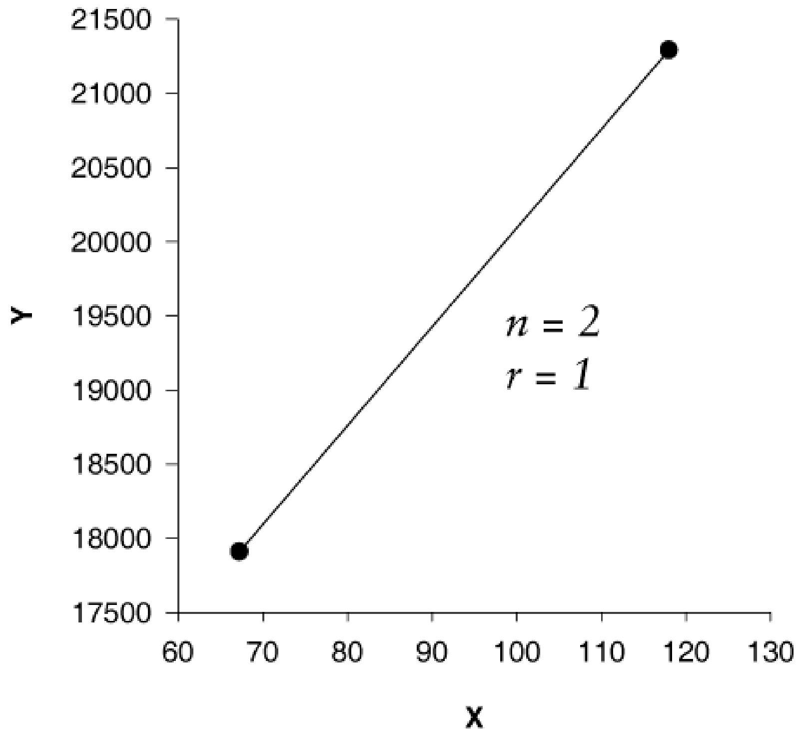




Roney et al. (2003)

Adjusted *r*

If we sample only 2 (x, y) points, we'll get a perfect correlation in the sample, r = 1 (or –1). But that doesn't mean that the correlation in the underlying population, ρ, is perfect! A better **estimator** of ρ is r_{adj}:



$$r_{adj} = \sqrt{1 - \frac{(1-r^2)(n-1)}{n-2}}$$

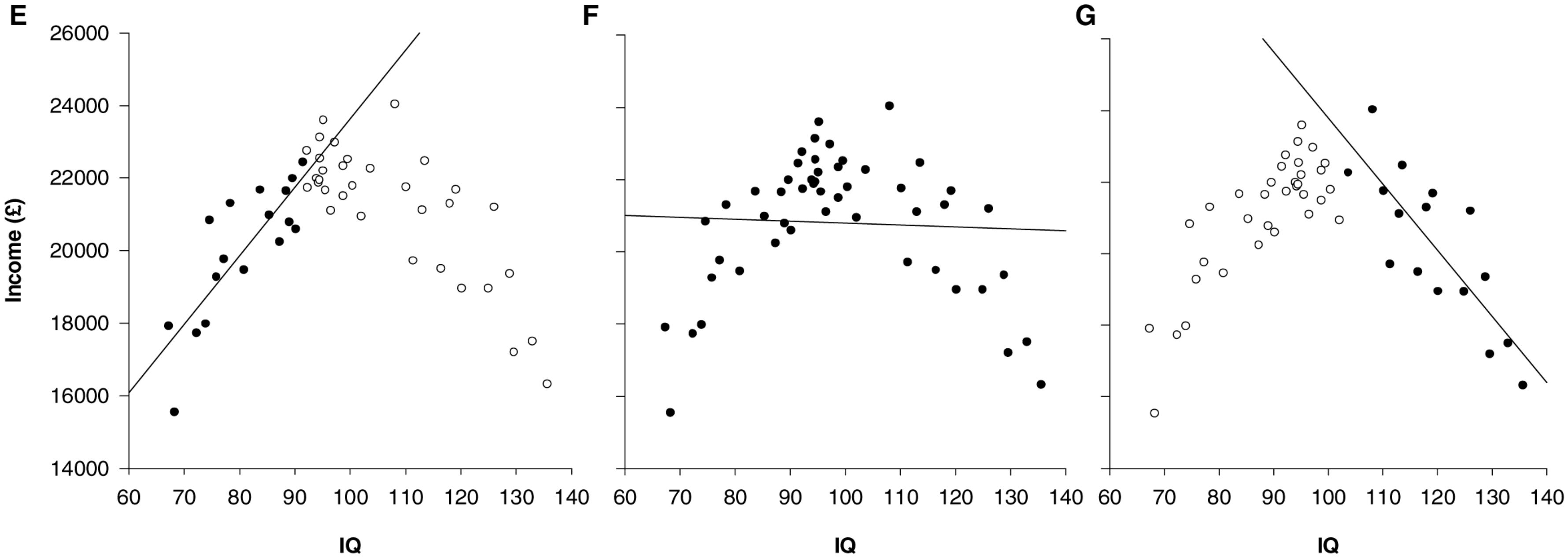
Adjusted *r*: mental rotation example

$$r = 0.954$$

$$n = 6$$

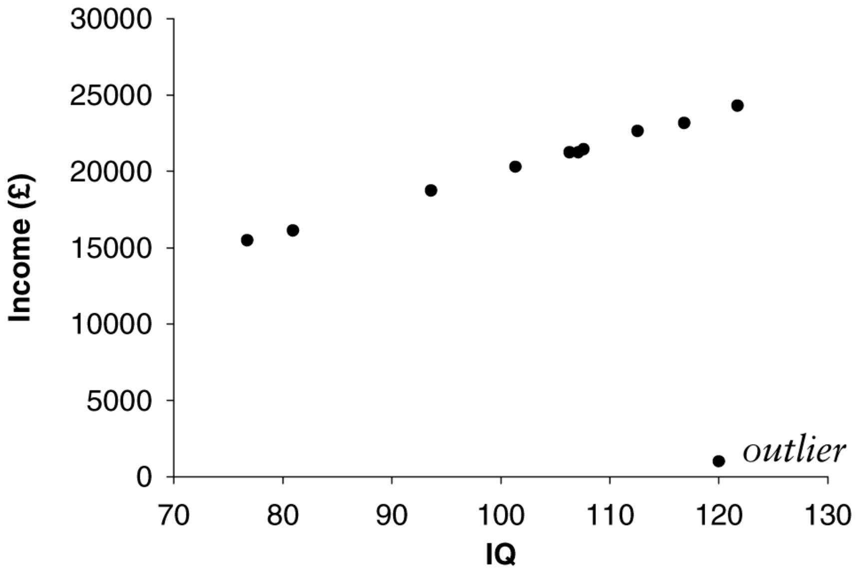
$$r_{adj} = \sqrt{1 - \frac{(1 - r^2)(n - 1)}{n - 2}} = 0.942$$

Beware of sampling a restricted range



Beware outliers!

H



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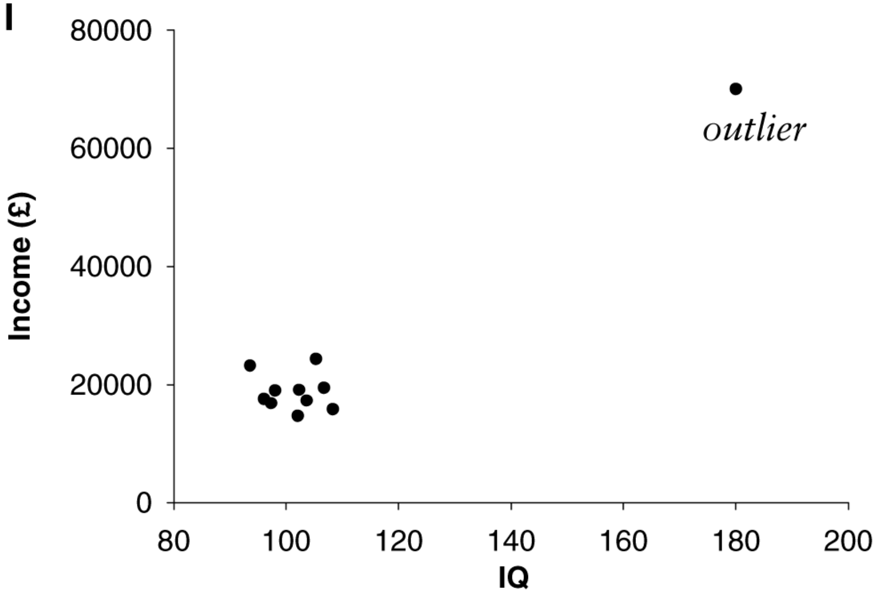
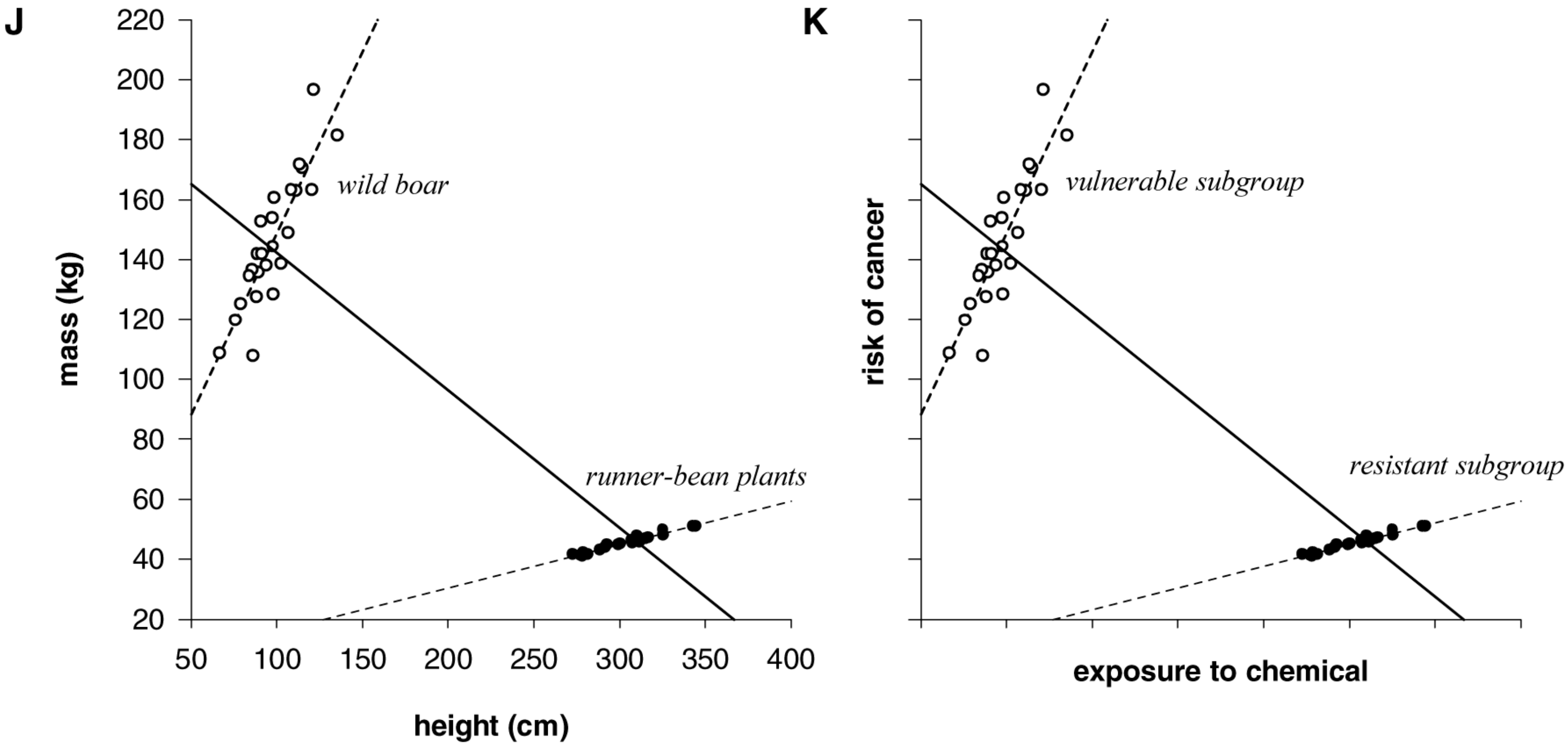


Heterogeneous subgroups: the oncologist and the magic forest



‘Is my correlation significant?’ Our first *t* test.

Research hypothesis: the correlation in the underlying population from which the sample is drawn is non-zero ($\rho \neq 0$). **Null hypothesis:** the correlation in the population is zero ($\rho = 0$). Calculate *t*:

$$t_{n-2} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

A big *t* (positive or negative) means that your data would be unlikely to be observed if the null hypothesis were true. Look up the **critical level** of *t* for **“*n*–2 degrees of freedom”** in the *Tables and Formulae*. Values of *t* near zero are not ‘significant’. If your |*t*| is more extreme than the critical value, it is significant. If your *t* is significant, you **reject** the null hypothesis. Otherwise, you don’t. We’ll explain *t* tests properly in the next practical.

t test: mental rotation example

$$r = 0.954$$

$$n = 6$$

$$t_{n-2} = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = 6.36$$

$$\text{critical } t_4 = 2.776 \text{ for } \alpha = 0.05 \text{ two tailed}$$

Assumptions you make when you test hypotheses about ρ

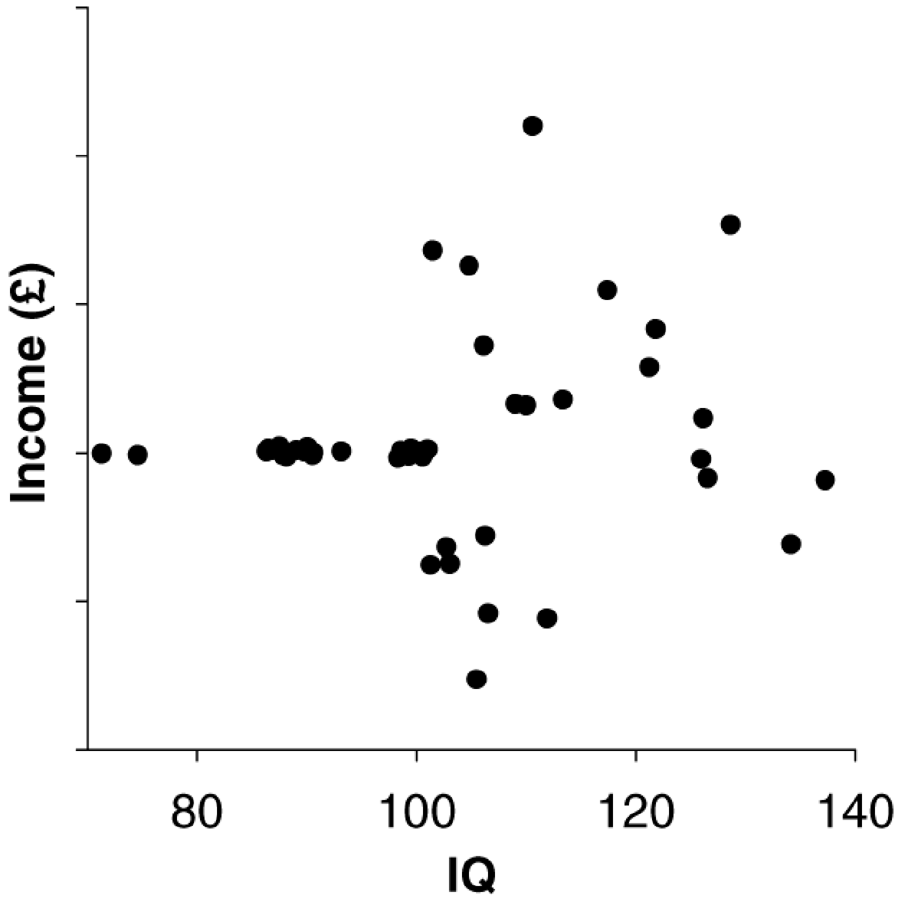
Basically, the data shouldn't look too weird. We must assume

• that the variance of *Y* is roughly the same for all values of *X*. (This is termed *homogeneity of variance* or *homoscedasticity*. Its opposite is *heterogeneity of variance* or *heteroscedasticity*.)

• that *X* and *Y* are both normally distributed

• that for all values of *X*, the corresponding values of *Y* are normally distributed, and vice versa

Heteroscedasticity is a Bad Thing (and Hard to Spell)



This would violate an assumption of testing hypotheses about ρ. So if you run the t test on r, the answer may be meaningless.

r_s: Spearman's correlation coefficient for **ranked** data

- Rank the *X* values.
- Rank the *Y* values.
- Correlate the *X* **ranks** with the *Y* **ranks.** (You do this in the normal way for calculating *r*, but you call the result r_s.)

- To ask whether the correlation is 'significant', use the table of critical values of Spearman's r_s in the *Tables and Formulae* booklet.

How to rank data

Suppose we have ten measurements (e.g. test scores) and want to rank them. First, place them in ascending numerical order:

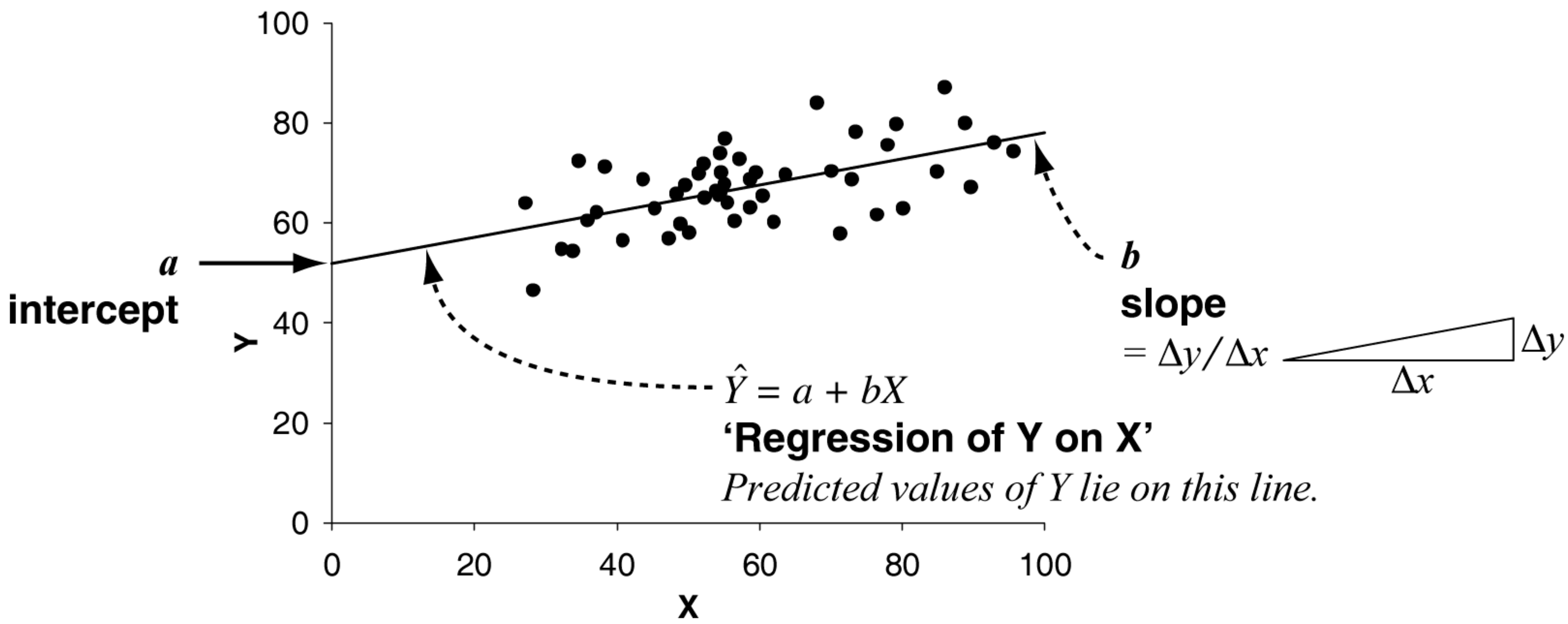
5	8	9	12	12	15	16	16	16	17

Then start assigning them ranks. When you come to a tie, give each value the mean of the ranks they're tied for — for example, the 12s are tied for ranks 4 and 5, so they get the rank 4.5; the 16s are tied for ranks 7, 8, and 9, so they get the rank 8:

X:	5	8	9	12	12	15	16	16	16	17
rank:	1	2	3	4.5	4.5	6	8	8	8	10

Regression

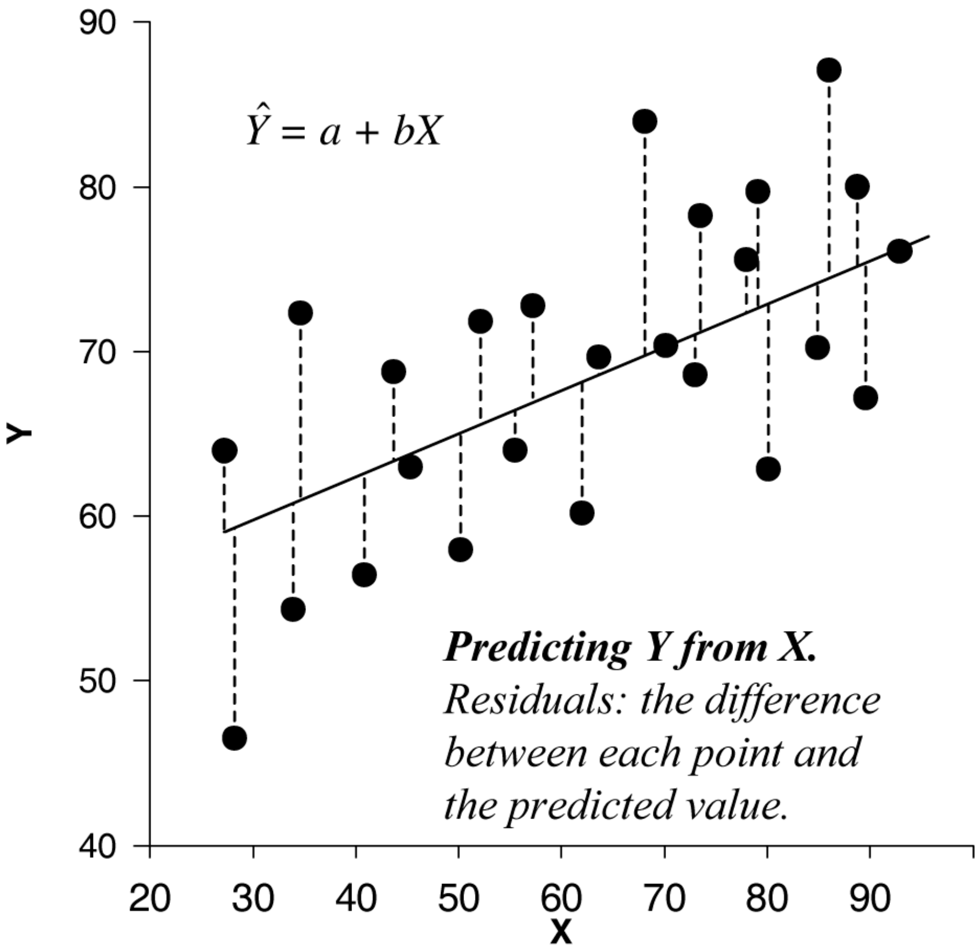
Linear regression: predicting things from other things



$$\hat{Y} = a + bX$$

But **which** line? **Which** values of *a* and *b*?

‘Least squares’ regression: finding *a* and *b*



Predicted value of *Y*:

$$\hat{y} = a + bx$$

Error in prediction **(residual):**

$$y - \hat{y}$$

Squared error:

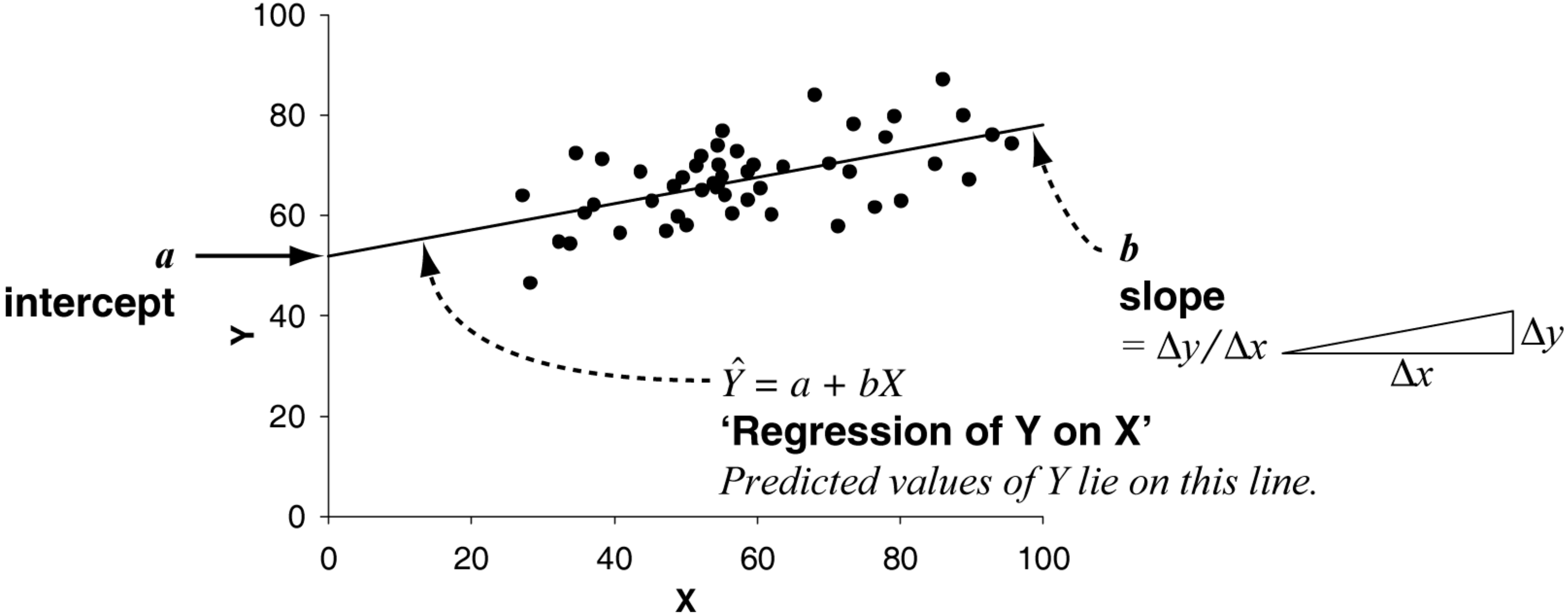
$$(y - \hat{y})^2$$

Sum of squared errors:

$$\sum (y - \hat{y})^2$$

We pick values of *a* and *b* in such a way that minimizes the sum of the squared errors.

We've found our best line.

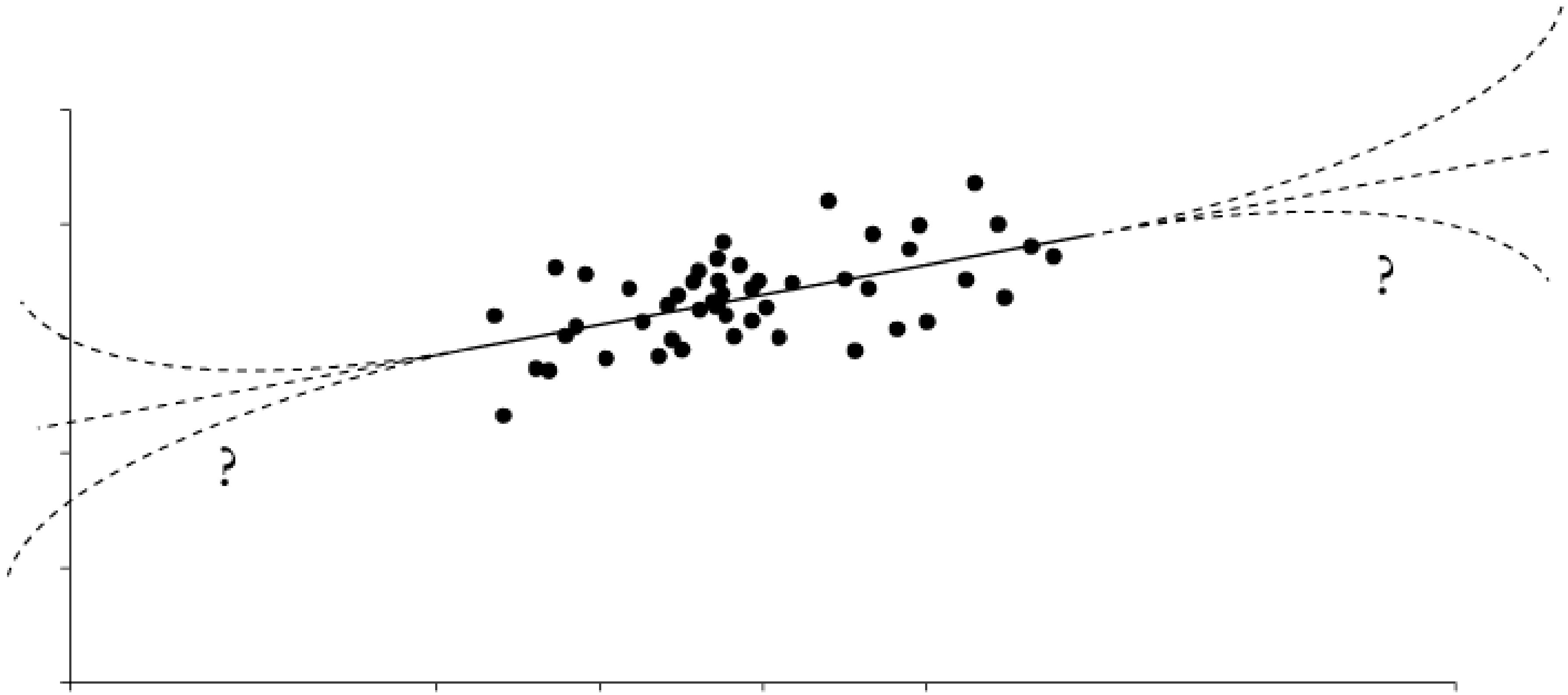


$$\hat{Y} = a + bX$$

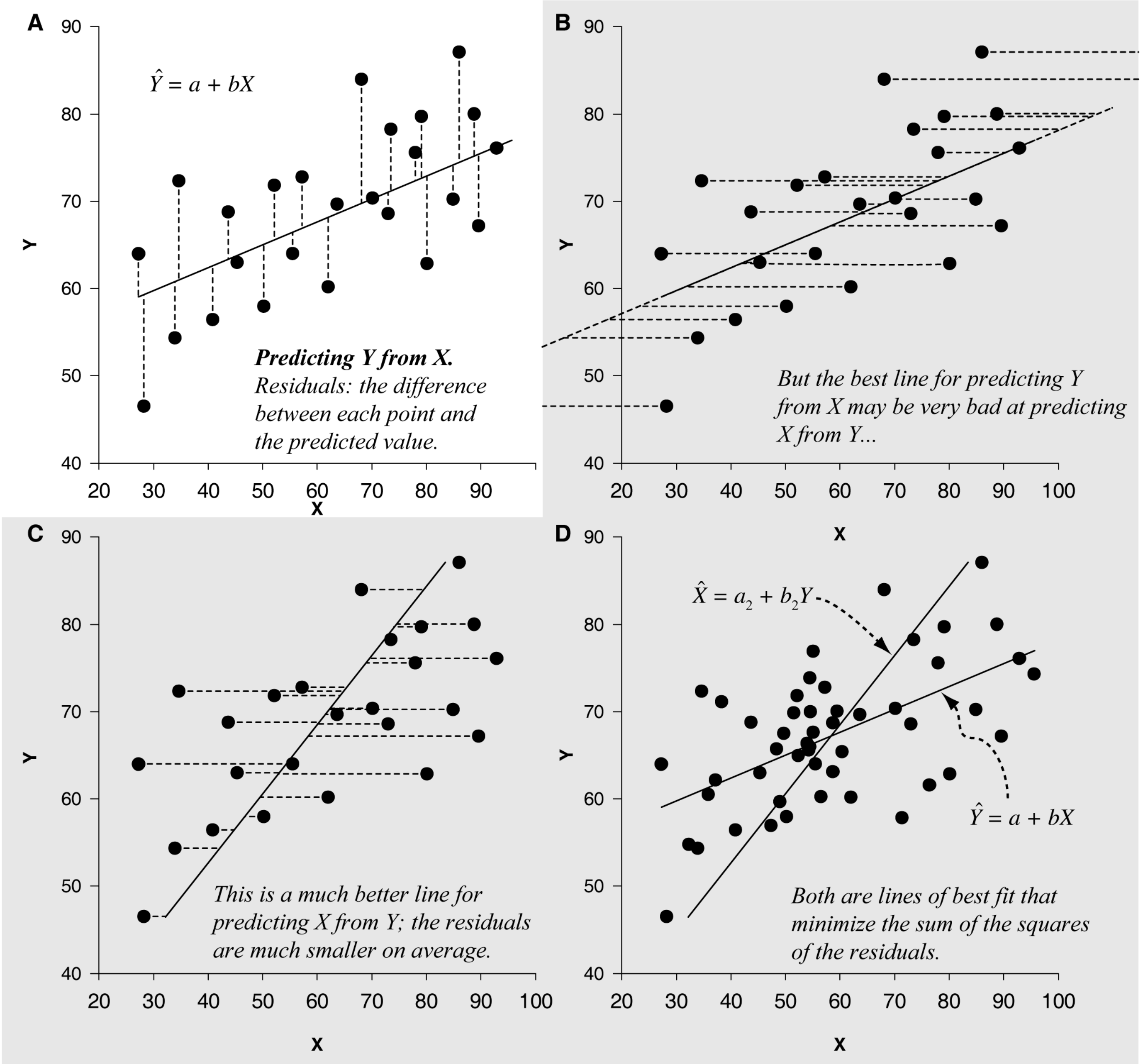
$$a = \overline{y} - b\overline{x} \qquad b = \frac{\text{cov}_{XY}}{s_X^2} = r\frac{s_Y}{s_X}$$

Our line will always pass through $(0, a)$ and $(\overline{x}, \overline{y})$.

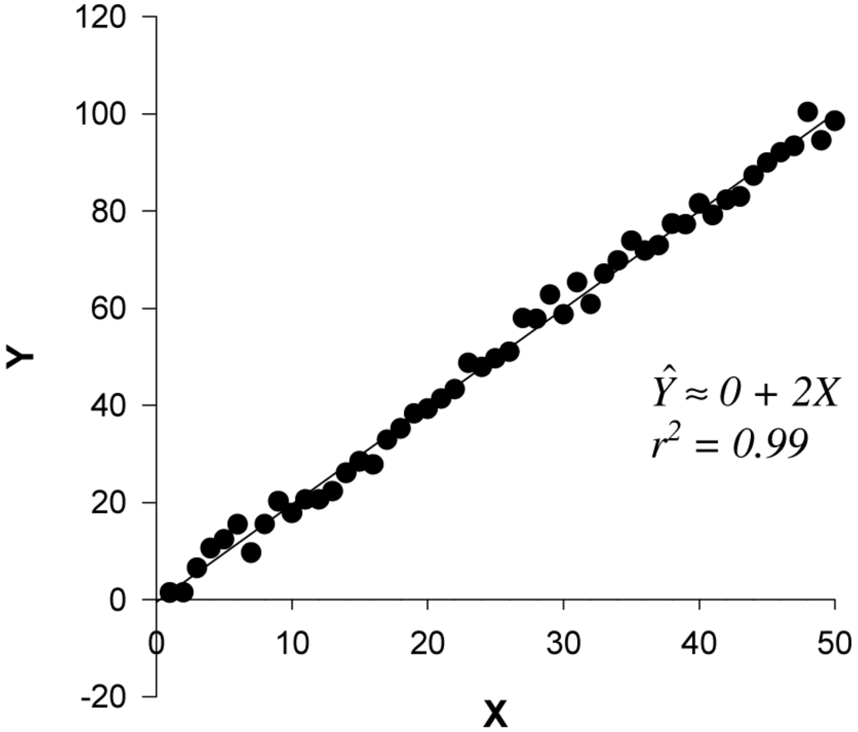
Beware extrapolating beyond the original data

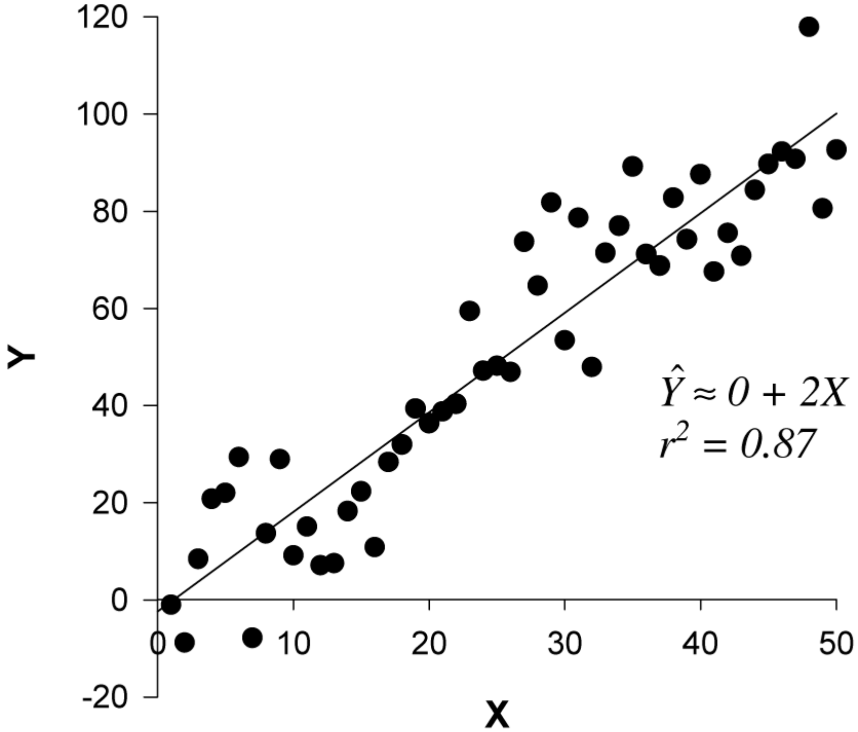


Predicting Y from X is **not** the same as predicting X from Y!



r^2 means something important — the proportion of the variability in Y predictable from the variability in X





Doing it for real

For **descriptive statistics** (mean, SD, etc.):

	Casio fx115s	Other Casio models
Enter descriptive statistics (SD) mode	MODE 2	MODE →SD
Clear the stats memory	SHIFT C (Scl)	SHIFT AC (Scl) =
Enter values of x (e.g. 53)	5 3 M+ (DATA DEL) etc.	
Read out descriptive statistics: *(see keypad and inside lid)* — mean	SHIFT 1 ($\bar{x}$)	SHIFT 1 ($\bar{x}$) =
sample SD (n–1 formula)	SHIFT 3 ($x\sigma_{n-1}$)	SHIFT 3 ($x\sigma_{n-1}$) =
n	RCL 3 ($x\sigma_{n-1}$)	RCL hyp (C)

For correlation and **linear regression** (r, a, b):

	Casio fx115s	Other Casio models
Enter linear regression (LR) mode	MODE 3	MODE →REG→Lin
Clear the stats memory	SHIFT C (Scl)	SHIFT AC (Scl) =
Enter values of x, y pairs (e.g. x = 53, y = 17)	5 3 [(--- (x_D, y_D) 1 7 M+ (DATA DEL) etc.	5 3 , 1 7 M+ etc.
Read out desired coefficients *(see keypad and inside lid)* — r	SHIFT 9 (r)	SHIFT ((r) =
a (A)	SHIFT 7 (A)	SHIFT 7 (A) =
b (B)	SHIFT 8 (B)	SHIFT 8 (B) =

shortest angle (°)	RT (ms)
0	830
60	908
120	1079
180	1387
120	1070
60	935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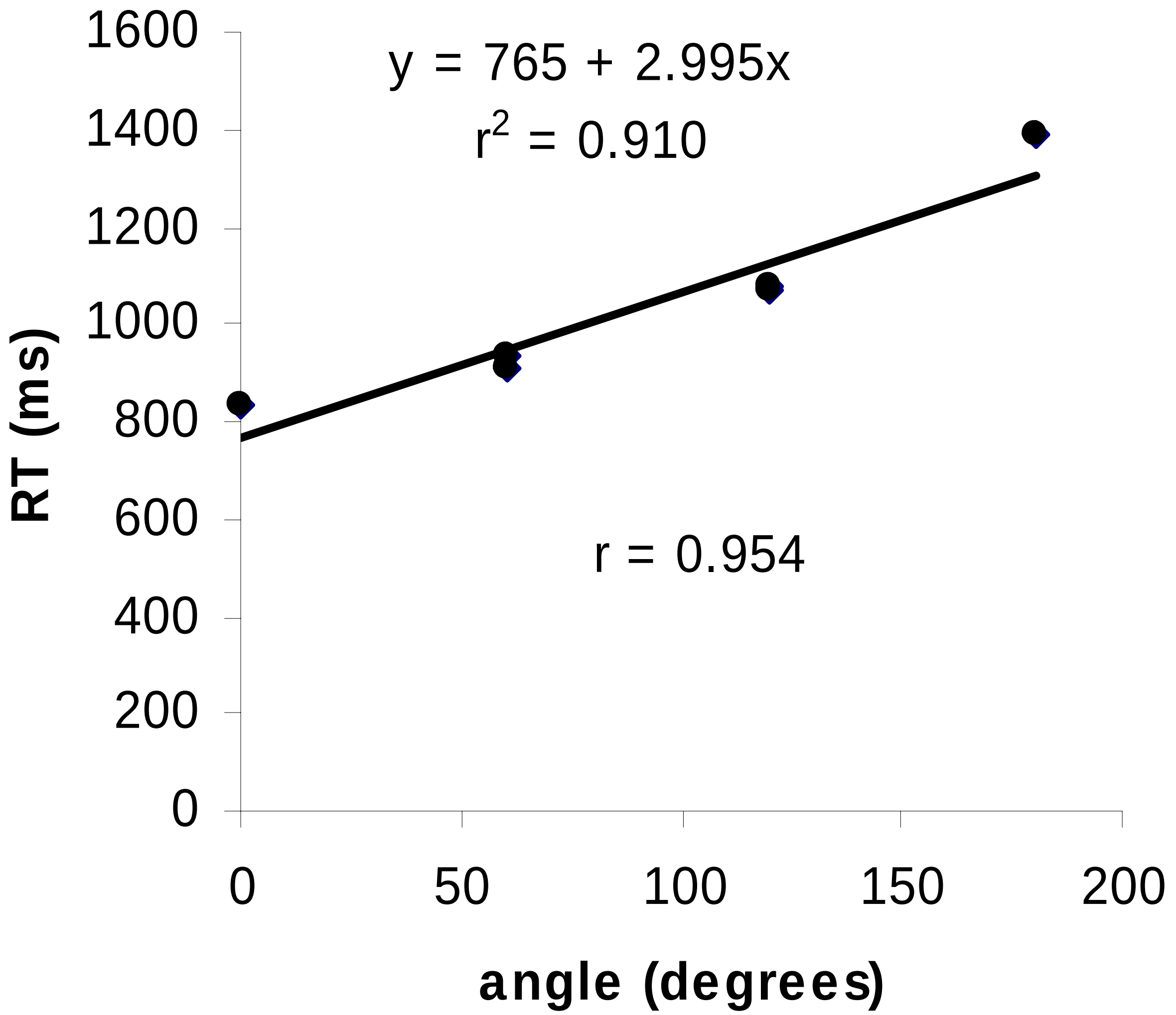
Mental rotation: regression line (group means, simplest task)



Thu 11 November 2004: Armistice Day